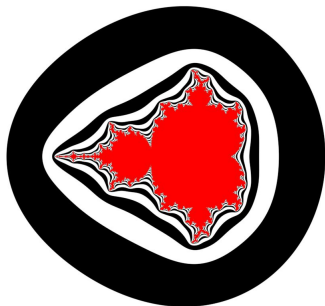


At the Helm of the Burning Ship

Claude Heiland-Allen

EVA London 2019

The Mandelbrot Set



$$x \rightarrow x^2 - y^2 + a$$

$$y \rightarrow 2xy + b$$

aka

$$z \rightarrow z^2 + c$$

The Burning Ship



$$x \rightarrow |x|^2 - |y|^2 + a$$

$$y \rightarrow 2|x||y| + b$$

Escape Time



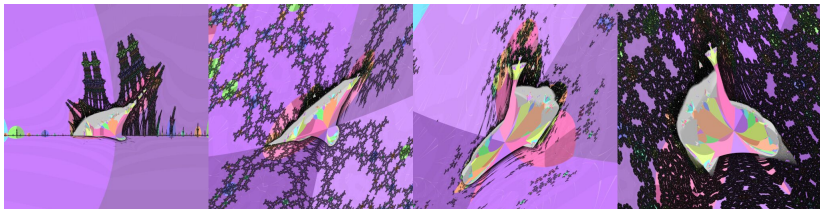
- ▶ count iterations until $\rightarrow \infty$
- ▶ smoothing: Vepstas (1997)

Distance Estimates



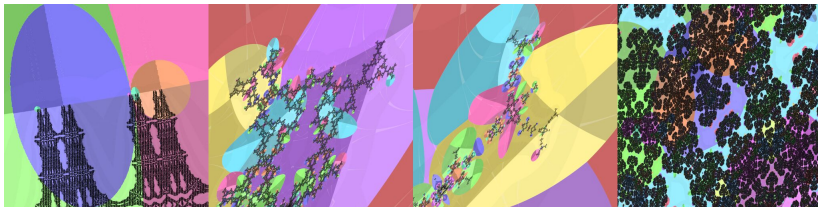
- ▶ bands pile up at boundary
- ▶ quantify
 - ▶ screen space differences
 - ▶ analytic derivatives
- ▶ colour: closer than a pixel?

Mini-ships



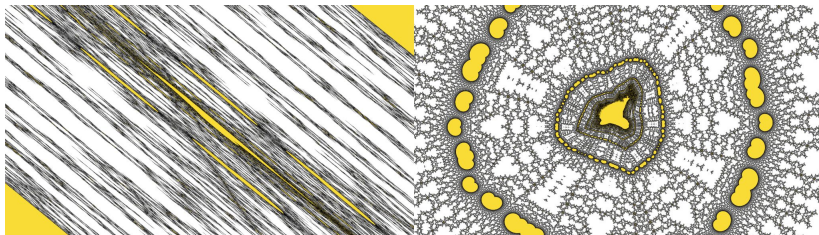
- ▶ periodic nucleus
- ▶ find period by iterating region
 - ▶ explicit polygon corners (Munafo 2008)
 - ▶ implicit derivatives (knighty 2017)
- ▶ find nucleus by Newton's method
- ▶ size estimate for Mandelbrot Set (Hunt & Ott 1997)

Misiurewicz Points



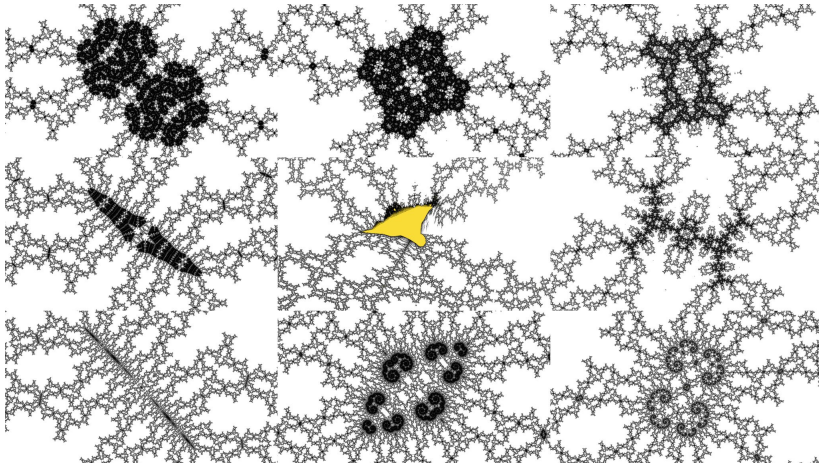
- ▶ pre-periodic point on boundary
- ▶ asymptotic self-similarity (spirals)
- ▶ multiplier zoom factor from derivative of cycle

Unskewing Stretched Areas



- ▶ regions under mini-ships are stretched
- ▶ polar decomposition of 2×2 derivative matrix
 - ▶ uniform scale
 - ▶ rotation
 - ▶ stretch amount
 - ▶ stretch angle

Embedded Julia Sets



- ▶ regular (non-embedded) Julia sets have
 - ▶ c constant across the whole image
 - ▶ z initialised to pixel coordinates

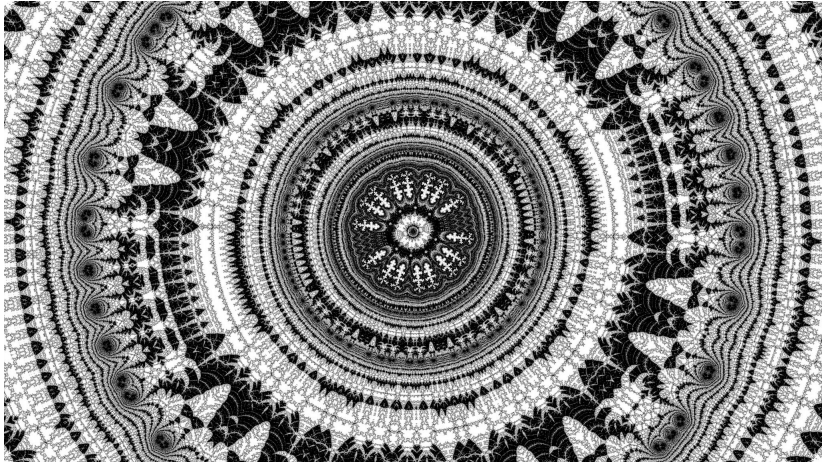
Perturbation

- ▶ **key observation: nearby pixels tend to remain nearby**
 - ▶ differences are small
 - ▶ orbit itself is large
 - ▶ symbolic algebra to avoid catastrophic loss of significance
- ▶ popularized by K.I. Martin's *SuperFractalThing* (2013)
- ▶ similar work by Sergey Khashin and Kruchenkova T. (2011)
- ▶ laser blaster's case analysis for $|X + x| - |X|$ (2014)
- ▶ Pauldelbrot's glitch detection criterion (2014)

Generalisations

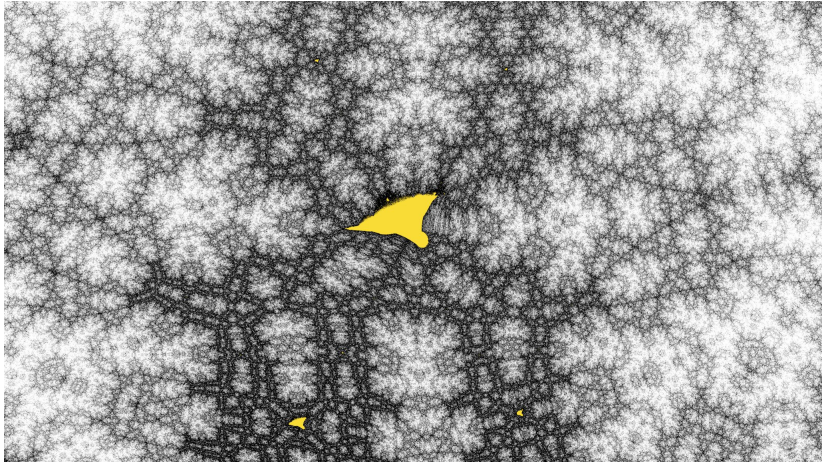
- ▶ techniques work for other formulas
 - ▶ Celtic, Buffalo, Mandelbar, . . .
 - ▶ higher powers
 - ▶ hybrids
- ▶ usually minor changes to the maths
- ▶ code generation can be automated

Shape Stacking



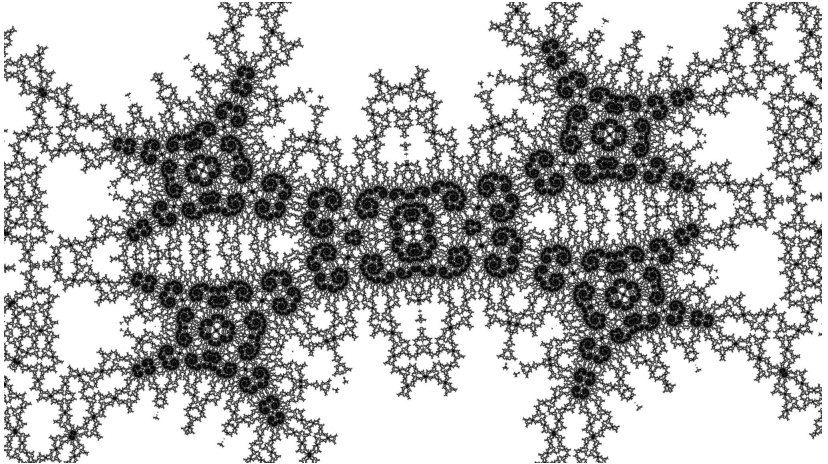
- ▶ zoom path gets recorded in the image
- ▶ history repeats, only twice as fast

Layer Stacking



- ▶ zoom to mini-ship in decorations
- ▶ focus on inner-most decorations

Julia Morphing



- ▶ zoom to embedded Julia set
- ▶ zoom off-centre in a specific way

Summary

- ▶ colourings highlight the structure of the fractal
- ▶ automate finding precise
 - ▶ coordinates
 - ▶ zoom depths
 - ▶ skew matrices

leaving more free time for artistic choices

- ▶ accelerate deep zooming many-fold
- ▶ applicable to a large class of fractals

Links

- ▶ <https://mathr.co.uk/et/>
et for Linux (experimental, with formula compiler)
- ▶ <https://mathr.co.uk/kf/kf.html>
Kalles Fraktaler 2+ for Windows (including WINE)
- ▶ <https://fractalforums.org>
artistic and research community
- ▶ <mailto:claudio@mathr.co.uk>